## CEMC at Home

Grade 11/12 - Monday, March 30, 2020 Check Your Calendar



#### You Will Need:

• Two players

#### How to Play:

- 1. Players alternate turns. Decide which player will go first (Player 1) and which player will go second (Player 2).
- 2. Player 1 starts the game by saying the date "January 1".
- 3. Player 2 then says a date later in the year which has either the same month or the same day as the date said by Player 1 ("January 1"). For example, Player 2 could say "January 5" (same month) or "March 1" (same day), but not "February 4" (different month and day).

Note: If you are playing with the 2020 calendar, then February 29 may be used!

- 4. The players now alternate saying dates, based on the date said previously, following the same rules as given in #3.
- 5. The player who says "December 31" wins the game!

### An example of a complete game:

Alexis and David are playing the game. They decide that Alexis will be Player 1.

Alexis: January 1
David: January 16
Alexis: April 16
David: July 25
Alexis: David!!! You can't change the month and day!
David: Oh yeah, right. July 16
Alexis: July 31
David: December 31! I win!

**Play this game a number of times.** Alternate which player goes first. Is there a winning strategy for this game? Does the winning strategy depend on whether you are Player 1 or Player 2?

Is there a connection between this game and the game we played on March 23 (Rook to the Top)?

### More Info:

Check out the CEMC at Home webpage on Monday, April 6 for a solution to Calendar Game. We encourage you to discuss your ideas online using any forum you are comfortable with.

### CEMC at Home

# Grade 11/12 - Monday, March 30, 2020 The Calendar Game - Solution

It turns out that the strategy for winning the Calendar Game is similar to the strategy for winning the game Rook to the Top (that we played on March 23). You might want to refresh your memory by having a look at the strategy for Rook to the Top.

In Rook to the Top, we played on an 8 by 8 grid. In some sense, we can also think of the Calendar Game as being played on a "grid". In this case it will be a 12 by 31 grid with some spaces not open for play.

December	1	2		19	20	21	22	 28	29	30	31
November	1	2		19	20	21	22	 28	29	30	
October	1	2		19	20	21	22	 28	29	30	31
September	1	2		19	20	21	22	 28	29	30	
	÷	:		:	:	:	1	3	3	:	:
March	:	:		: 19	: 20	: 21	: 22	 : 28	: 29	: 30	: 31
March February	: 1 1	: 2 2		: 19 19	: 20 20	: 21 <mark>21</mark>	: <mark>22</mark> 22	 : 28 28	: 29 29	: 30	: 31

Notice that the grid has a row for each of the 12 months, and the rows contain either 29, 30 or 31 squares, depending on how many days are in that particular month (in 2020). The diagonal that is highlighted on the grid is the one we will focus on for the winning strategy of this game. We will refer to it as the *winning diagonal*.

The winning diagonal consists of the following dates:

January 20	July 26
February 21	August 27
March 22	September 28
April 23	October 29
May 24	November 30
June 25	December 31

Since Player 1 must say "January 1", we see that the first "move" lands off of the winning diagonal. Player 2 can then "move" onto the winning diagonal by saying "January 20". Now Player 1 must change either the month or the day (but not both) and so any allowed date will represent a move off of the winning diagonal. If they change the month, this corresponds to a vertical move upwards and if they change the day, this corresponds to a horizontal move to the right. Player 2 can now choose the appropriate date from the table above to move back onto the winning diagonal. For example, if Player 1 changes the month and says "May 20", Player 2 can then change the day and say "May 24" (from the table above). If instead Player 1 changes the day and says "January 27", Player 2 can then change the month and say "August 27" (from the table above).

Repeating this process, Player 1 will always have to move off of the winning diagonal, and Player 2 will always be able to return to the winning diagonal, closer to December 31. Since there are a finite number of dates to choose from, Player 2 will eventually say December 31.

Thus, Player 2 has a winning strategy for this game.