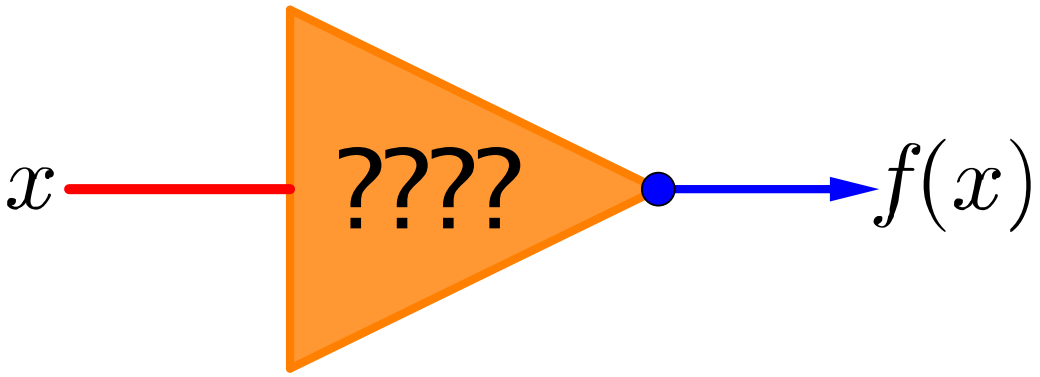
CEMC at Home features Problem of the Week

Grade 11/12 - Thursday, March 26, 2020

Functionally Possible

The function $f(x) = x^5 - 3x^4 + ax^3 - x^2 + bx - 2$ has a value of 5 when $x = 3$.

Determine the value of the function when $x = -3$.

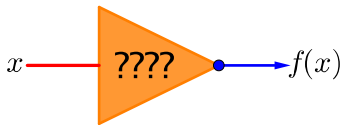


More Info:

Check the CEMC at Home webpage on Thursday, April 2 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, April 2.

This CEMC at Home resource is the current grade 11/12 problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week and to find many more past problems and their solutions visit: https://www.cemc.uwaterloo.ca/resources/potw.php



Problem of the Week
Problem E and Solution
Functionally Possible

Problem

The function $f(x) = x^5 - 3x^4 + ax^3 - x^2 + bx - 2$ has a value of 5 when $x = 3$.

Determine the value of the function when $x = -3$.

Solution

We know that the function has a value of 5 when $x = 3$. Therefore, $f(3) = 5$.

$$\begin{aligned} f(3) &= 5 \\ (3)^5 - 3(3)^4 + a(3)^3 - (3)^2 + b(3) - 2 &= 5 \\ 243 - 243 + 27a - 9 + 3b - 2 &= 5 \\ 27a + 3b &= 16 \qquad (1) \end{aligned}$$

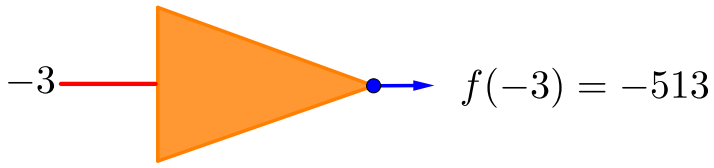
At this point we seem to have used up the given information. Maybe we can learn more by looking at precisely what we are asked to determine.

In this problem, we want the value of the function when $x = -3$. In other words, we want $f(-3)$.

$$\begin{aligned} f(-3) &= (-3)^5 - 3(-3)^4 + a(-3)^3 - (-3)^2 + b(-3) - 2 \\ &= -243 - 243 - 27a - 9 - 3b - 2 \\ &= -27a - 3b - 497 \end{aligned}$$

But from (1) above, $27a + 3b = 16$ so
$f(-3) = -27a - 3b - 497 = -(27a + 3b) - 497 = -16 - 497 = -513$.

Therefore, the value of the function is -513 when $x = -3$.



We are not given enough information to find the precise values of a and b but enough information is given to solve the problem.