



CEMC at Home

Grade 11/12 - Monday, March 23, 2020

Rook to the Top

Do you have a chessboard at home? Get it out, grab another person and let's play a game! If you don't know how to play chess, don't worry!

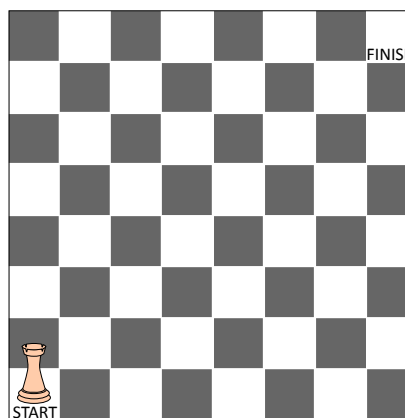
You Will Need:

- Two players
- A chessboard or checkerboard

If you can't find a board, then you can draw an 8×8 grid on a piece of paper.

- A rook (as shown)

If you can't find a rook, then you can use a coin or any small object in place of the rook.



How to Play:

1. Place the rook in the bottom left corner of the board.
2. The two players will alternate turns moving the rook. Decide which player will go first.
3. On your turn, you can move the rook as many squares as you want either to the right or up.
You must move the rook at least one square and you cannot move the rook both right and up on the same turn. And of course you cannot run the rook off the board!
4. The player to place the rook in the top right corner of the board wins the game!

Play this game a number of times. Alternate which player goes first. Is there a winning strategy* for this game? Does the winning strategy depend on whether you move the rook first or second?

* A *strategy* is a pre-determined set of rules that a player will use to play the game. The strategy dictates what the player will do for every possible situation in the game. It's a *winning strategy*, if the strategy allows the player to always win, regardless of what the other player does.

Variation:

- Cover up the bottom 3 rows of the chessboard and start with the rook in the new bottom left corner. Play the game with the same rules. Does this change the winning strategy?

More Info:

Check out the CEMC at Home webpage on Monday, March 30 for a discussion of a strategy for this game. We encourage you to discuss your ideas online using any forum you are comfortable with.

We sometimes put games on our math contests! Check out [Question 2](#) on the 2003 Hypatia Contest for another game where we are looking for a winning strategy.

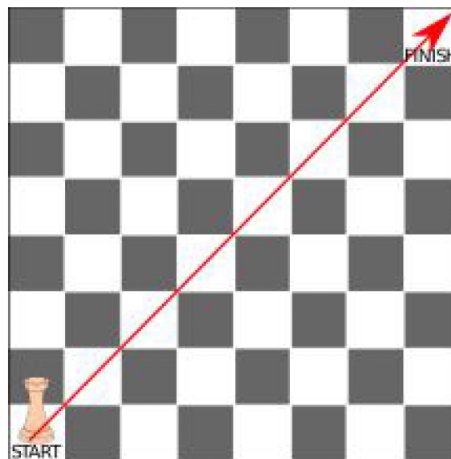


CEMC at Home

Grade 11/12 - Monday, March 23, 2020

Rook to the Top - Solution

We are going to call the diagonal line of white squares indicated in diagram, the *main diagonal*. Playing this game, you probably realized that the main diagonal is important to the strategy of this game.



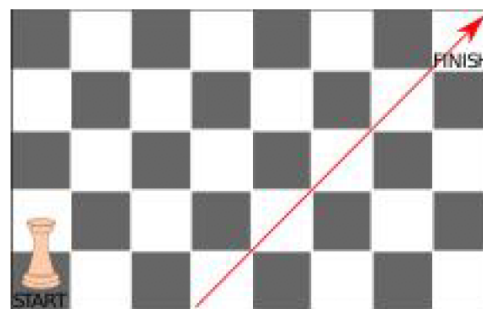
The rook begins on the main diagonal. The first player moves the rook and no matter what move they make, they will have to move the rook off of the main diagonal. If the first player moves the rook n squares to the right, then the second player can move the rook n squares up and the rook will be back on the main diagonal. If the first player moves the rook n squares up, then the second player can move the rook n squares to the right and the rook will be back on the main diagonal. In such a way the second player can guarantee that the rook will be on the main diagonal after their turn and the rook will be closer to the top right square (and maybe even at this square)!

Since the rook is back on the main diagonal, the first player must again move the rook off of the main diagonal and the second player can again put it back on to the main diagonal. Repeating this process, the second player will always be able to place the rook on the main diagonal closer to the top right square. Since there are a finite number of squares on the chessboard, the second player will eventually place the rook in the square at the top right corner.

Thus, we can see that the second player has a winning strategy for this game.

Variation:

In the variation of this game, we have a board with only five rows. We refer to the diagonal shown as the main diagonal. In this variation, the rook does not start on the main diagonal. If the first player moves the rook three spaces to the right, the rook will then be on the main diagonal. After this first move, the second player has no choice but to move the rook off of the main diagonal, leaving the first player the opportunity to place it back on the diagonal. Then the strategy continues as described for the first version. Therefore, the first player has the winning strategy in this variation of the game.



Extension: Consider a chessboard with any number of rows and any number of columns. For what size of chessboard will the first player have a winning strategy? For what size of chessboard will the second player have a winning strategy?